

Parent Handbook for Mathematics, English Version



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A Message from the State Superintendent of Public Instruction

We live in a world of mathematics. Every day millions of people use numbers as an integral part of their lives. We plan household budgets, figure the mileage on our cars, calculate sales tax, or measure the space in a living room. We may tabulate the receipts from a local fund-raiser or calculate the cost of food for the office picnic. Using a higher level of mathematics, we design highways and bridges, build stadiums, plan the acoustics for concert halls, and explore outer space.

Perhaps no other subject in school better teaches students the processes of logic and abstract thinking. Life requires us to contend with unknown quantities and variables and to "square" certain facts that, at first appearance, might seem to be contradictory. Mathematics trains students to be analytic and provides them with a foundation for intelligent and precise thinking.

Sometimes referred to as "the queen of sciences," mathematics spans almost every facet of life, encompassing everything from the most mundane, everyday tasks to the wonder and order of the universe. It makes understandable what otherwise would be imponderable. It makes possible the accomplishment of routine activities and the expansion of scientific knowledge. We cannot conceive of a world in which mathematics is not fundamental and essential.

For these reasons California's mathematics curriculum is explicit and rigorous. It is based upon the premise that all students are capable of learning mathematics and learning it well. With persistent effort, effective instruction, and support from home, students not only will develop fluency in computational skills but also will be able to understand mathematical concepts, solve problems, find solutions, reason mathematically, and communicate with precision. They will be well prepared for the demands of everyday life, the workplace, and participation as knowledgeable citizens.

In this parent handbook you will learn about the mathematics curriculum for California public schools. You will learn about the state standards at each grade level and find information on statewide testing and graduation requirements. Perhaps what is most important, you will gain ideas for helping your child to be successful in this subject, for success is best achieved when schools and families work together.

I hope you and your family find this handbook to be a valuable resource.

JACK O'CONNELL
State Superintendent of Public Instruction

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The Importance of Mathematics

Mathematics has long been recognized as one of the most essential subjects in education. The reason for this recognition becomes obvious when we think of how often mathematics is used and how many situations there are in which mathematics means the difference between success or failure, life or death. For example, a wedding cake will be made large enough to feed a hundred guests only if the baker's calculations have been correct; a family will be able to make payments on a refrigerator or buy shoes and clothes for the children only by budgeting the family's income; and the safety of an aircraft depends on the mathematical calculations of aeronautical engineers. All human endeavors—agriculture, industry, trades, technology, law, sciences, crafts, and arts, for example—use this amazing branch of study in various ways.

All students need to develop proficiency in mathematics. Students must learn how to think logically and solve meaningful, challenging problems. They must understand the depth and breadth of mathematical concepts and perform simple, complex, and abstract calculations accurately.

Even if students do not pursue careers that demand the study of advanced mathematics, they will find that knowledge of mathematics is essential for success in many occupations and professions. With an understanding of mathematics, students can consider numerous career options. They must have access to high-quality

mathematics programs that give them opportunities to choose from a full range of future career paths. Although this ideal will not be easy to achieve, to strive for less would be a disservice to California's students and to society.

A solid grasp of mathematics gives students competence and self-assurance in the practical matters of everyday life: balancing a checkbook, purchasing a car, managing a food budget, serving as treasurer for a club or other organization, or understanding the daily news. Further, they will recognize the interdependence of mathematics and music, art, architecture, science, philosophy, and other disciplines and will become lifelong students of mathematics regardless of the careers they pursue.

Proficiency in most of mathematics is not an inborn characteristic; it is a consequence of persistence, effort, practice, support, encouragement, and effective instruction. Today, the challenges for parents and educators are to expect more from students in mathematics, raise the bar for student achievement through a more rigorous curriculum and instruction, and provide the support necessary for all students to meet or exceed those expectations. The mastery of a challenging subject area allows students to gain confidence and satisfaction in learning.

In this handbook parents will find numerous suggestions for home or family activities that can help students succeed in mathematics. They will also find an overview of California's mathematics curriculum and the mathematics requirements for high school graduation and college/university admission. The final section of the handbook offers sample resources that support students' mathematics education at home and in school.

How You Can Help Your Children Succeed in Mathematics

Educational research indicates that families are an essential part of the learning process. By doing mathematics with your children and supporting mathematics learning at home, you contribute greatly to your children's success.

There are many ways in which parents can make mathematics part of family life. Parents and teachers, through the Sonoma County Office of Education and the North Bay Math Project, contributed many of the suggestions and activities offered here. As you establish your own practices in supporting what your children are learning, you may consider the following ideas:

A. Always talk about mathematics in positive ways.

Regardless of your own background in mathematics, be positive when you discuss mathematics with your children. Your encouragement will help ensure that they do not develop mental blocks about the subject. Work with your children to overcome trouble spots. Let them know how important mathematics is by pointing out how people use mathematics in everyday life.

B. Have high expectations for your children.

Be confident that your children can learn mathematics and then actively support them as they do so. Seek out mathematics-focused programs and activities for your children. As they get older, encourage them in their study of algebra and other courses. Encourage students to take as many advanced courses in high school as possible so that they will be prepared for the choices ahead of them.

C. Give young children a good start in mathematics.

The seeds of many important mathematics concepts are planted when children are young, and early experiences can determine how a child looks at mathematics for the rest of his or her life. It is never too early to start learning mathematics.

Between the ages of two and four, children generally experience mathematics through simple counting. Counting is a basic and important concept that helps children bring order to the world around them. Early counting and "how many" experiences (for example, "How many fingers am I holding up? How many crackers do you have?") introduce children to mathematics concepts that become deeper and more complex when they are in elementary school. For example, counting three dimes later becomes a way of understanding 30 cents. The more opportunities young children have to count, the better they will understand the meaning and use of numbers—and the more confidence they will have with mathematics later on.

Parents of young children may find the following tips helpful:

- Encourage children to count frequently. Find things to count every day, everywhere, and in every way. Start slowly with just a few things. As your children's ability to count grows, find bigger and bigger collections of different things for them to count.
- Encourage children to count real objects: cookies, coins, toys, and so forth. Children discover that counting is more than a singsong repetition when they count real objects. Encourage children to say one number as they touch each object. Arrange objects in different ways for counting—for example, in piles, rows, and circles. Have the children count blocks as they build a tower.
- Reinforce your children's counting. When your children finish counting, you can say, "One, two, three cookies. You counted three cookies. You have three cookies!" To correct a mistake, gently count again along with your children. Make counting a game you and your children enjoy—and play often.
- Do not worry if children use their fingers. Fingers are the best mathematical tools children have prior to learning to write numbers—and they are always handy and ready to use!

When your children become proficient at counting, teach them how to count by twos, fives, and even tens. Remember that counting is fundamental—a great start for learning mathematics.

D. Make mathematics a part of your family's everyday activities.

Spend time with your children on simple board games, puzzles, and activities centered on mathematics. Involve them in activities such as shopping, cooking, or home fix-it projects to show them that mathematics is practical and useful. By pointing out the mathematics in everyday life, you can help your children learn some basic concepts and understand why mathematics is so important. Provide assistance when necessary, but let the children figure things out by themselves. Find ways to make mathematics fun.

Here are a few ideas for helping children discover—and use—the mathematics around them.

In a play area young children can:

- Sort toys by size, kind, or color.
- Put dolls, cars, or blocks in order from largest to smallest.
- Play What Am I Thinking Of? by describing a toy's size and shape.
- Play make-believe store with toys and favorite objects.

In the kitchen young children can:

- Look for familiar two-dimensional shapes—circles, squares, triangles, and so forth—such as a round pot or a square napkin.
- Put cans of food in order by size or type.
- Sort silverware from the dishwasher to the drawer.
- Count plates, utensils, cups, or even olives.
- Divide cookies evenly so that every family member gets an equal share.
- Find how many glasses of milk are in a full milk carton.
- Help double a recipe or reduce it by half.

Around the house children of various ages can:

- Count the days on a calendar until a special event is to occur.
- Find the length and width of a room by pacing it off.
- Draw a diagram of how to rearrange furniture in a room.
- Make a map of the whole house. Older students may draw their maps to scale.
- Create a family TV schedule and track the time devoted to watching TV.
- Calculate or track the statistics of favorite athletes.
- Help an adult by calculating the number of yards of cloth needed to make curtains or clothes.

When they are outside, young children can:

- Set up and operate a lemonade stand under adult supervision.
- Plant a garden with rows and columns of seeds.
- Count the petals on different flowers.
- Measure a sunflower or bean plant daily to keep track of its growth.
- Count how many times they can jump rope or shoot baskets in a row.
- Keep a daily chart of temperatures.
- Find triangles, squares, circles, and rectangles around the house or in the neighborhood.

When traveling, older children can:

- Use the scale on a map to calculate the approximate number of miles to a destination.
- Total the daily costs of a trip (for food, mileage, bus or train tickets, for example) and then determine the average daily expenses after the trip is completed.

E. Take opportunities to play math games with your children.

Many games provide enjoyable ways to reinforce the mathematics that children learn at school; for example, board games involving counting or the exchange of play money; card games requiring the tallying of points or the collection of cards in suits, kinds, or sequences; and models that require measurement and computation in order to assemble them. These are just a few of the many possible examples. Games provide not only pleasant ways for adults and children to use math together but also opportunities for older children to guide and coach younger children in applying mathematics.

F. Notice mathematics in the world at large.

Mathematics is everywhere. It is in the world that surrounds us: in nature, in the city, in the country, and even in outer space. You can help your children see the usefulness of mathematics if you point it out to them wherever you see it.

Tell your children about the mathematics you do in your job and its importance to your work. When you are outside your home, look for ways to point out mathematics: What shape does that tree look like? What is the shape of a baseball playing field, and what is the distance between bases? By counting the number of musicians in the first row of a marching band and then counting the number of rows, children can determine the total number of players in the band. If you start looking for examples, you will see more and more mathematics everywhere. And so will your children.

G. Become familiar with what your children are studying in mathematics.

Ask the children what they are studying in mathematics class, talk with them about mathematics homework, and help with school projects when it is appropriate. Talk with your children's teachers; feel free to ask them about the mathematics standards for your children's grade levels.

H. Help your children to use technology appropriately in mathematics.

Within the past 20 years, new technology has become a central part of everyday life. Many people in widely different occupations and vocations find that computers, calculators, and the Internet can be useful and valuable tools in their work.

Although children need to know how to use technology in their education, it is essential that they first understand mathematics and develop the ability to use mathematics effectively *without* technology. Children must have the chance to practice basic computation and mathematical processes over an extended period of time.

When students take the state tests in mathematics under the Standardized Testing and Reporting (STAR) program, they are not allowed to use a calculator. And the

State Board of Education-adopted mathematics standards for California public schools do not mention technology until grade six. (Information about state standards and the state STAR tests may be found in the later sections of this handbook.)

Once students are ready to use calculators and computers, however, technology provides a practical tool not only for solving certain kinds of problems but also for broadening students' opportunities and interests in mathematics. For example, students solving tasks involving long and complex calculations, charts, tables, graphs, and spreadsheets learn the power of using mathematics and technology together. If students do not have computers at home, teachers at school libraries and some classrooms may make computers available for student use.

Helpful information regarding the use of technology in the mathematics curriculum may be found in Chapter 9 of the *Mathematics Framework for California Public Schools*, which may be accessed online at <http://www.cde.ca.gov/ci/ma/cf/index.asp>.

In addition, the booklet *Math at Home: Helping Your Children Learn and Enjoy Mathematics*, published by the Sonoma County Office of Education, contains useful suggestions for the use of technology by students. Parents may refer to page 9 of the booklet, which may be accessed online at http://www.scoe.org/files/math_english.pdf.

I. Help your child at home with mathematics homework.

As a parent you will probably want to help your child with homework—but you may not feel prepared to help with mathematics, especially as children get older and the mathematics becomes more complex. If so, you may be pleased to learn that you do not have to be an expert to help with your child's homework.

Providing a well-lighted table and comfortable chair is an important place to start. The best location for homework is not necessarily your son's or daughter's bedroom. If your child studies at the kitchen or dining room table instead, you can help without having to sit down the entire time. You can assist when needed and still go about your own tasks. At the same time, you have the opportunity to keep homework time focused by giving your child support, encouragement, and gentle reminders.

A good strategy is to pass by your child's work area and periodically "check in." A quick glance will often tell you if it is time to stop and provide some extra support. When it is clear that your child could use some assistance, take a deep breath and relax. If you stay calm, it will help your child stay calm.

Many parents worry about not knowing the mathematics their child is studying. In this case the way to provide homework help is actually quite simple: ask questions and practice careful listening. Simple generic questions can help a child gradually make sense of mathematics, build confidence, and encourage mathematical thinking and

communication. When given the opportunity to talk about mathematics, children are often able to remember what they learned in class and see the solution themselves. A good question can open up a child's thinking about the problem at hand.

Remember that listening to your child's answers and providing calm responses are as important as the questions you pose. When a child is unsure as to how to begin a problem, ask:

- Can you tell me what you know now? What facts do you have?
- What do you need to find out? Can you estimate the answer?
- Can you show me an example of this kind of problem in your book?
- How might you begin? What can you try first?
- Can you make a drawing or picture to get started?

While your child is working on a problem, ask:

- How can you organize your information? Will a list or table help?
- What would happen if . . . ? Show me what you did that didn't work.
- Can you explain the strategy you're using to solve this? Why did you . . . ?
- What could you do next? Do you see any patterns?

When your child finds an answer, ask:

- How did you get your answer? Do you think it's right? Tell me more.
- Explain why your solution makes sense.

When questions alone will not do, another strategy for helping your child is to identify a friend or relative who knows more mathematics than you do. Find out if that person would be willing to answer an occasional phone call from your son or daughter.

The Internet can be a helpful resource because some Web sites offer homework assistance with mathematics. Librarians or library/media teachers can assist parents and children in exploring the following:

The Math Forum's Ask Dr. Math: <http://mathforum.org/dr.math/>

Tutor.com: <http://www.tutor.com>

J. Seek assistance when your child experiences difficulty in mathematics.

If your child is having difficulty in a mathematics class, talk to the teacher, school counselor, or principal. They can recommend strategies to assist your student or provide resources that your child may find helpful. Parents might also want to explore after-school programs, summer school, and tutoring options as well as print and Web resources designed to provide students with assistance.

Additional suggestions for parents and children may be found in the booklet *Math at Home: Helping Your Children Learn and Enjoy Mathematics*, produced by the Sonoma County Office of Education and available online at www.scoe.org/files/math_english.pdf.

For samples of mathematics resources, both in print and on the Internet, see “Sample Resources for Parents and Students” at the back of this handbook.

STANDARDS AND FRAMEWORKS: What They Are and What Role They Play

State law requires the California State Board of Education to establish policies for each major subject area taught in public schools. Typically, the State Board sets forth these policies in the form of *content standards* and *curriculum frameworks*. These two terms appear prominently in discussions about public school curriculum and are fundamental in determining what students should learn and teachers teach; therefore, a brief explanation is provided here.

Content standards are written expectations of what all students at a given grade level should know and be able to do. The expectations are high; they are comparable to the academic standards of countries that have high levels of student achievement.

Content standards define the skills and knowledge that students need in order to be literate, educated citizens and to enter a college or university. The standards also constitute the basis of statewide tests that students must take at certain grade levels. District and school administrators, classroom teachers, universities that prepare teachers, and publishers of textbooks and other instructional materials pay close attention to the content standards in their work.

A curriculum framework describes the content of the course for each grade level, kindergarten through grade twelve, and offers suggestions to teachers on *how to* teach the curriculum. A framework is a kind of blueprint for implementing the content standards adopted by the State Board of Education. Many teachers and administrators use a framework as a guide to help them coordinate what they will teach. Local school boards sometimes base their own curriculum decisions on the frameworks adopted by the State Board. Many teacher education programs use frameworks as a source for professional learning. A framework also informs textbook publishers of the kinds of instructional materials needed in schools.

The State Board's content standards and framework for mathematics outline a rich program of studies for all children in public schools. The standards are based on the premise that all students are capable of learning and using rigorous mathematic skills, concepts, and tools. In addition, the standards ensure that students at the same grade level learn similar concepts and skills regardless of what public school they attend in California.

An overview of the mathematics curriculum appears in the next section, but parents are welcome to read California's mathematics standards and framework in their entirety. Both the *Mathematics Content Standards for California Public Schools, Kindergarten Through Grade Twelve* and the *Mathematics Framework for California*

Public Schools, Kindergarten Through Grade Twelve (2006 edition) may be viewed online at <http://www.cde.ca.gov/ci/ma/cf>.

In addition, copies of both publications are available for purchase from CDE Press. For prices and ordering information, interested parents may e-mail CDE Press at sales@cde.ca.gov or call (800) 995-4099.

“All students are capable of understanding mathematics, given the opportunities and encouragement to do so.”

—*Mathematics Content Standards for California Public Schools*

An Overview of the Mathematics Curriculum

The California mathematics standards for kindergarten through grade twelve have changed the teaching and learning of mathematics. The mathematics curriculum at each grade level is designed to ensure that every student has access to high-quality mathematics programs and opportunities to build a firm foundation in mathematics.

The curriculum begins with basic mathematics and keeps growing in scope and content as the years progress. Mathematics education must provide students with a balanced instructional program in which they gradually develop an understanding of concepts, proficiency in basic computational skills, and the ability to solve problems.

The goal in mathematics education is for students to:

- Develop fluency in basic computational skills.
- Develop an understanding of mathematical concepts.
- Become mathematical problem solvers who can recognize and solve routine problems readily and can find ways to reach a solution or goal where no routine path is apparent.
- Communicate precisely about quantities, logical relationships, and unknown values through the use of signs, symbols, models, graphs, and mathematical terms.
- Develop logical thinking in order to gather data, analyze evidence, and build arguments to support or refute hypotheses.
- Make connections among mathematical ideas and between mathematics and other disciplines.
- Apply mathematics to everyday life and develop an interest in pursuing advanced studies in mathematics and a wide array of related career choices.
- Develop an appreciation for the beauty and power of mathematics.

Kindergarten Through Grade Seven

In kindergarten through grade seven, the mathematics standards are divided into five topics or strands:

- Number Sense
- Algebra and Functions
- Measurement and Geometry
- Statistics and Probability
- Mathematical Reasoning

Each of the strands consists of many mathematics concepts. Students engage in deeper, more sophisticated learning within each strand as they advance through the grade levels. Each set of standards at each grade level includes all five strands.

The following information describes state guidelines on mathematics at each grade level. The complete set of standards for each grade level can be found in the *Mathematics Content Standards for California Public Schools*, which is available on the CDE Web site at <http://www.cde.ca.gov/ci/ma/cf> or in print from the California Department of Education, CDE Press Sales Office (call 1-800-995-4099 for price and ordering information).

Kindergarten

By the end of kindergarten, students understand small numbers, quantities, and simple shapes in their everyday environment. They count, compare, describe and sort objects, and develop a sense of properties and patterns.

Grade One

By the end of grade one, students understand and use the concept of ones and tens in the place value number system. Students add and subtract small numbers with ease. They measure with simple units and locate objects in space. They describe data and analyze and solve simple problems.

Grade Two

By the end of grade two, students understand place value and number relationships in addition and subtraction and use simple concepts of multiplication. They measure quantities with appropriate units. They classify shapes and see relationships among them by paying attention to the elements that compose them (what mathematics teachers call *geometric attributes*). Students collect and analyze data and verify answers.

Grade Three

By the end of grade three, students deepen their understanding of place value and their understanding of and skill with addition, subtraction, multiplication, and division of whole numbers. Students estimate, measure, and describe objects in space. They use patterns to help solve problems. They represent number relationships and conduct simple probability experiments.

Grade Four

By the end of grade four, students understand large numbers and addition, subtraction, multiplication, and division of whole numbers. They describe and compare simple fractions and decimals. They understand the properties of, and the relationships between, plane geometric figures. They collect, represent, and analyze data to answer questions.

Grade Five

By the end of grade five, students increase their facility with the four basic arithmetic operations applied to fractions, decimals, and positive and negative numbers. They know and use common measuring units to determine length and area and use formulas to determine the volume of simple geometric figures. Students know the concept of angle measurement and use a protractor and compass to solve problems. They use grids, tables, graphs, and charts to record and analyze data.

Grade Six

By the end of grade six, students have mastered the four arithmetic operations with whole numbers, positive fractions, positive decimals, and positive and negative integers; they accurately compute and solve problems. They apply their knowledge to statistics and probability. Students understand the concepts of mean, median, and mode of data sets and how to calculate the range. They analyze data and sampling processes for possible bias and misleading conclusions; they use addition and multiplication of fractions routinely to calculate the probabilities for compound events. Students conceptually understand and work with ratios and proportions; they compute percentages (e.g., tax, tips, interest). Students know about *pi* and the formulas for the circumference and area of a circle. They use letters for numbers in formulas involving geometric shapes and in ratios to represent an unknown part of an expression. They solve one-step linear equations.

Grade Seven

By the end of grade seven, students are adept at manipulating numbers and equations and understand the general principles at work. Students understand and use factoring of numerators and denominators and properties of exponents. They know the Pythagorean theorem and solve problems in which they compute the length of an unknown side. Students know how to compute the surface area and volume of basic three-dimensional objects and understand how area and volume change with a change in scale. Students make conversions between different units of measurement. They know and use different representations of fractional numbers (fractions, decimals, and percents) and are proficient at changing from one to another. They increase their facility with ratio and proportion, compute percents of increase and decrease, and compute simple and compound interest. They graph linear functions and understand the idea of slope and its relation to ratio.

Grades Eight Through Twelve

After studying the five strands of mathematics from kindergarten through grade seven, students concentrate on specialized areas of mathematics in greater depth in grades eight through twelve. *Note:* The mathematics standards for grades eight through twelve are organized by discipline rather than by strand:

- Algebra I
- Geometry
- Algebra II
- Trigonometry
- Mathematical Analysis
- Linear Algebra
- Probability and Statistics
- Advanced Placement Probability and Statistics
- Calculus

Mathematical reasoning and conceptual understanding are included in the content of every mathematics discipline that students study at more advanced levels.

The following information describes state guidelines in the various mathematics disciplines:

Algebra I

Symbolic reasoning and calculations with symbols are central to algebra. Through the study of algebra, a student develops an understanding of the symbolic language of mathematics and the sciences. In addition, algebraic skills and concepts are developed and used in a wide variety of problem-solving situations.

The key content for the first course, Algebra I, involves understanding, writing, solving, and graphing linear and quadratic equations, including systems of two linear equations in two unknowns. Quadratic equations can be solved by factoring, completing the square graphically, or applying the quadratic formula. Students should also become comfortable with operations on monomial and polynomial expressions. They learn to solve problems by employing all of these techniques and thereby extend their mathematical reasoning in many important ways, including justifying steps in an algebraic procedure and checking algebraic arguments for validity.

Geometry

The main purpose of the geometry curriculum is to develop geometry skills and concepts and the ability to construct formal logical arguments and proofs in a geometry setting. Although the curriculum stresses plane (synthetic) Euclidean geometry, there is room to place special emphasis on coordinated geometry and its transformations.

The geometry skills and concepts developed in this discipline are useful to all students. Aside from learning these skills and concepts, students will develop their

ability to construct formal, logical arguments and proofs in geometric settings and problems.

Algebra II

This discipline complements and expands the mathematical content and concepts of Algebra I and Geometry. It introduces many new concepts and techniques that will be basic to more advanced courses in mathematics and the sciences and useful in the workplace. Generally, abstract thinking skills, the function concept, and the algebraic solution of problems in various content areas are emphasized.

Students who master Algebra II will gain experience with algebraic solutions of problems in various content areas, including the solution of systems of quadratic equations, logarithmic and exponential functions, the binomial theorem, and the complex number system.

Trigonometry

Trigonometry uses the techniques that students have previously learned from the study of algebra and geometry. The trigonometric functions studied are defined geometrically rather than in terms of algebraic equations. Facility with these functions as well as the ability to prove basic identities regarding them is especially important for students intending to study calculus, more advanced mathematics, physics and other sciences, and engineering in college.

Mathematical Analysis

This discipline combines many of the trigonometric, geometric, and algebraic techniques needed to prepare students for the study of calculus and strengthens their conceptual understanding and mathematical reasoning in solving problems. These standards take a functional point of view toward those topics. The most significant new concept is that of limits. Mathematical analysis is often combined with a course in trigonometry or perhaps with one in linear algebra to make a yearlong precalculus course.

Linear Algebra

The general goal in this course is for students to learn the techniques of matrix manipulation so that they can solve systems of linear equations in any number of variables. Linear algebra is most often combined with another subject, such as trigonometry, mathematical analysis, or precalculus.

Probability and Statistics

This discipline is an introduction to the study of probability, interpretation of data, and fundamental statistical problem solving. Mastery of this academic content will provide

students with a solid foundation in probability and facility in processing statistical information.

Probability and Statistics, Advanced

This discipline is a technical and in-depth extension of probability and statistics. In particular, mastery of academic content for advanced placement gives students the background to succeed in the Advanced Placement examination in the subject. (For information regarding Advanced Placement, see the section titled “Planning for Success: Requirements and Decisions.”)

Calculus

When taught in high school, calculus should be presented with the same level of depth and rigor as are entry-level college and university calculus courses. These standards outline a complete college curriculum in one variable calculus. Many high school programs may have insufficient time to cover all of the content in a typical academic year. For example, some school districts may treat differential equations lightly and spend substantial time on infinite sequences and series. Others may do the opposite. Consideration of the College Board syllabi for the Calculus AB and Calculus BC sections of the Advanced Placement Examination in Mathematics may be helpful in making curricular decisions. Calculus is a widely applied area of mathematics and involves a beautiful intrinsic theory. Students mastering this content will be exposed to both aspects of the subject.

District Options for Grades Eight Through Twelve

School districts, schools, and teachers have flexibility in teaching these high school disciplines. The standards for grades eight through twelve do not require a particular course to be initiated or completed in any particular grade. Moreover, the subject matter for these various disciplines is sometimes combined differently. For example, some schools teach the standards in traditional mathematics courses, such as Trigonometry and Algebra II. Other schools teach the standards in an integrated or a blended fashion. That is, one school might combine some trigonometry, mathematical analysis, and linear algebra content to form a precalculus course; another school might offer trigonometry content together with Algebra II.

Planning for Success: Requirements and Decisions

A. The Standardized Testing and Reporting Program

California law mandates statewide testing of certain subjects taught at specific grade levels. The purpose of the legislation is to determine student achievement at the state, county, school district, school, and individual student level. Called the Standardized Testing and Reporting (STAR) program, the testing program consists of the California Standards Tests (CSTs); the California Alternate Performance Assessment (CAPA); and the California Modified Assessment (CMA), and the Standards-based Test in Spanish (STS).

The California Standards Tests

All students in grades two through seven are required to take the CST for mathematics, which is based on grade-level mathematics standards. From grades eight to eleven, students take the CST corresponding to the courses in which they are enrolled. For example, students enrolled in Algebra I take the CST in Algebra I, students enrolled in geometry take the CST for geometry, and so on. In addition to the Algebra I and geometry CSTs, there are CSTs in Algebra II, general mathematics, high school summative mathematics, and integrated mathematics. Students in grades eight and nine not enrolled in a specific mathematics course take the general mathematics CST. Students who have completed Algebra II or Integrated Mathematics III take the high school summative mathematics test.

The California Alternate Performance Assessment

The CAPA is designed for students with significant cognitive disabilities who are unable to take the CSTs even with accommodations or modifications. Participation in the CAPA is specified in the student's individualized education program. Included in the CAPA are mathematics tasks that measure the students' achievement of the mathematics standards selected for those students.

The California Modified Assessment

The CMA is designed to assess those students whose disabilities preclude them from achieving grade-level proficiency on an assessment of the California content standards with or without accommodations. The CMA has been developed to provide more access so students can better demonstrate their knowledge of the California content standards. Students shall have taken the CST in a previous year and scored below basic or far below basic in the subject area being assessed by the CMA and may have taken the CST with modifications.

The Standards-based Tests in Spanish

The STS are standards-based tests developed exclusively for California public schools. These tests are administered to Spanish-speaking English learners who either receive instruction in their primary language or have been enrolled in a school in the United States fewer than 12 months. The STS are based on California content standards in reading-language arts and mathematics which allows Spanish-speaking English learners to demonstrate achievement of the California content standards.

Teachers, parents, and students can use individual test results to help monitor a student's progress. Additional information about the STAR program may be found on the California Department of Education Web site at <http://www.cde.ca.gov/ta/tg/sr>.

B. High School Graduation Requirements

Courses that incorporate California's mathematics standards are rigorous and demanding. The standards are designed to prepare students for the challenges they will face in a career and in higher education, both of which routinely require a comprehensive knowledge of mathematics.

You can help your children become successful in mathematics by understanding what they are required to learn and by stressing the importance of hard work. Be sure they know of your confidence in their ability to succeed in mathematics. Continue to speak positively about mathematics and actively support their learning.

If your child is experiencing difficulty in a mathematics course, discuss the problems with the teacher, school counselor, or principal. They can recommend strategies to assist your student or provide helpful resources. Parents might also want to explore tutoring, after-school programs, summer school, or print and Web resources designed to provide student assistance.

Two years of mathematics, including Algebra I, is required for graduation from a California high school. If a student completes Algebra I in grade seven or grade eight, he or she still must take two additional years of mathematics to meet state graduation requirements. School districts may add other requirements to the state minimum requirements; therefore, parents and students should confer with teachers and counselors.

Algebra I Requirement

Beginning in 2003-04 state law requires that students satisfactorily complete a course or combination of courses meeting the state content standards for Algebra I in order to graduate from high school. Because algebra is

considered the foundation of all higher-level mathematics and is essential for many careers, students must master the subject to advance to other classes and graduate from high school. To ensure that students succeed in high school algebra, schools increasingly emphasize instruction in the beginning concepts of algebra throughout the elementary and middle school years.

Does every student need algebra? Many researchers and teachers indicate that all students, regardless of their prior mathematical skills, benefit from taking algebra. Therefore, every student should be prepared to study algebra and be provided with opportunities to study algebra under high-quality teachers with access to the resources and support systems that will promote student success. Algebra is the passport to advanced mathematics, success in college and university, and success in tomorrow's careers. Without a fundamental understanding of algebra, students may be unable to succeed in completing the courses needed for admission to institutions of higher education.

Algebra builds on arithmetic to develop more sophisticated tools to complete tasks, such as determining which is the most cost-effective telephone plan or how much to invest at a particular interest rate to earn a certain amount. Algebra can also instill logical thinking and persistence in students. The U.S. Department of Labor reports that the highest-paying, fastest-growing job sectors all require significant technical skills. Algebra is the language of mathematics, which itself is the language of the information age. Every effort must be made to keep the doors of opportunity open for everyone to study algebra.

Advanced Placement Courses

If students are highly successful in mathematics, they may be offered opportunities to take Advanced Placement (AP) mathematics courses in high school. Students who take the highest level of mathematics study available in high school can earn college credits. Taking AP mathematics courses provides a distinct advantage for college-bound students. Applicants who excel in completing these courses are given special consideration by colleges and universities. Teachers of mathematics and counselors can provide information to students or parents regarding AP courses and eligibility requirements.

California High School Exit Examination

California high school students, beginning with the graduating class of 2006, must pass the California High School Exit Examination (CAHSEE) and meet all other state and local graduation requirements to receive a high school diploma. Students who do not pass the exam in grade ten will have opportunities to take it again in grades eleven and twelve. The purpose of the examination is to improve student achievement in high

school and help ensure that students who graduate from high school demonstrate competency in the state's content standards for reading, writing, and mathematics.

CAHSEE has two parts: English–language arts and mathematics. The mathematics portion is designed to test students' grasp of the mathematics topics outlined in grade seven standards through the first half of the algebra I standards. The test includes statistics, data analysis and probability, number sense, measurement and geometry, mathematical reasoning, and algebra. Students must demonstrate strong computational skills and a foundation in arithmetic, including an ability to work with decimals, fractions, and percentages. The mathematics portion of the exam consists entirely of multiple-choice questions.

More information about CAHSEE, including sample problems, may be available at your local high school. The California Department of Education offers additional information on the Internet at <http://www.cde.ca.gov/ta/tg/hs>.

C. University Admission Requirements

In considering university requirements, parents and students should begin planning while the student is in middle school and should discuss admission requirements with high school counselors. They should also refer to the following Web site for information on admissions to the University of California (UC) and California State University (CSU): <http://www.californiacolleges.edu>.

A minimum of three years of approved college preparatory mathematics courses is required for admission as a freshman into the UC or the CSU system. (Four years of mathematics is recommended, especially for the UC system.) The three approved courses must include at least Algebra I, Geometry, and Intermediate Algebra.

The minimum eligibility requirements for freshman admission to the UC system, in addition to a high school diploma, are the subject requirement, the scholarship requirement, and the examination requirement. Students meet the scholarship requirement by attaining a minimum grade point average in required subjects. All freshman applicants must also submit their scores on the SAT 1 or the American College Test as well as three SAT II Subject Tests, including Mathematics Level 1c or 2c.

Courses required by the UC are sometimes referred to as the "a–g" requirements because of the order in which the requirements are listed. Students and parents can view by high school the university-approved courses that help to meet those requirements by visiting the Web site <https://doorways.ucop.edu/list/>.

Sample Resources for Parents and Students

Whether in school or at home, students can enjoy and understand mathematics concepts more readily by reading good books. By combining mathematics and children's literature, parents can offer young students a learning experience that touches both worlds. Students who are gaining proficiency at counting, for example, enjoy such books as Wanda Gag's *Millions of Cats* and Mitsumasa Anno's *Anno's Counting Book*. Later, mathematics-related literature can help students understand the relationship of mathematics to other school subjects. For example, *The Doorbell Rang*, by Pat Hutchins, offers an example of scarcity, an economics concept commonly explored in history–social science during the primary grades. And David Bodanis's *E=mc²: A Biography of the World's Most Famous Equation* helps high school students learn how mathematics relates to world history and energy, mass, and light.

Parents are encouraged to accompany their children on trips to public or school libraries. Librarians or library/media teachers can help locate literature relating to mathematics. In addition, parents and teachers are encouraged to peruse online the California Department of Education publication *Literature for Science and Mathematics, Kindergarten Through Grade Twelve* for a number of mathematics-related titles. This extensive bibliography is helpful in identifying age-appropriate books available in libraries and bookstores. The list may be accessed at <http://www.cde.ca.gov/ci/cr/rl/>.

Some county offices and school districts have developed lists of literature that complement the study of various subjects. Parents can consult with library/media staff in school districts or county offices to find the resources available in their vicinity.

Other Printed Materials

A number of publishers have produced helpful resources that range from activity books and instructional materials to official state publications.

Algebra to Go: A Mathematics Handbook. Wilmington, Mass.: Great Source Education Group, 2000. This small handbook is a handy reference tool for students and parents. It provides short definitions, examples, and lessons on the concepts students study from grade five through algebra I.

Family Math, by Jean Stenmark, Virginia Thompson, and Ruth Cossey. Berkeley: Lawrence Hall of Science, University of California, Berkeley Press, 1999. *Family Math* is a popular book containing dozens of mathematics activities that parents and children ages eight to twelve can do together. Included are activities related to number

sense, geometry, probability and statistics, and algebra. A Spanish version of the book, *Matemática para la familia*, is also available.

Family Math for Young Children, by Grace Coates and Jean Kerr Stenmark. Berkeley: Lawrence Hall of Science, University of California, Berkeley Press, 1999. A sequel to *Family Math*, this book was developed for families with children ages four through eight.

Family Math—The Middle School Years, by Virginia Thompson and Karen Mayfield-Ingram. Berkeley: Lawrence Hall of Science, University of California, Berkeley Press, 1998. The activities suggested in this book cover algebraic reasoning and number sense and are appropriate for students in grades six through eight.

Helping Your Child Learn Math, by Patsy Kanter and Linda Darby. Washington, D.C.: U.S. Department of Education, 1999. This booklet offers advice for parents, suggested resources, and mathematics activities for children in kindergarten through grade six. It is available online at <http://www.ed.gov/pubs/parents/Math>.

Helping Your Child with Homework. Sacramento: California Department of Education, 1995. This publication answers questions that parents of children in elementary and junior high school often ask about homework. Some ideas may also be helpful for high school students. For prices and ordering information, contact the publisher at sales@cde.ca.gov or call (800) 995-4099.

Math at Home: Helping Your Children Learn and Enjoy Mathematics. Santa Rosa, Calif.: Sonoma County Office of Education, 2002. This public service publication has served as a major source of the information included in this handbook. In addition to suggesting activities for home and study, the booklet lists nearly 40 selections of children's literature that support mathematics in kindergarten through grade four. Copies of the booklet may be obtained through the Sonoma County Office of Education, Curriculum and Instruction Division. Telephone (707) 524-2600 or go online at http://www.scoe.org/files/math_english.pdf.

Math Matters: Kids Are Counting on You. PTA Kit. Washington, D.C.: National Academy Press, 1998. PTAs and other parent groups that want to help teachers and administrators bring mathematics to the forefront of their elementary school will find this kit helpful. A Spanish version is also available.

Math on Call: A Mathematics Handbook, by Andrew Kaplan et al. Wilmington, Mass.: Great Source Education Group, 2004. Short definitions, examples, and lessons on more than 300 mathematics concepts studied in kindergarten through grade eight are included in this small handbook for students and parents.

Mathematics-Related Web Sites

With the growth of the Internet, a number of mathematics-related Web sites have become available. The following sites may also prove helpful to parents, students, and teachers:

Figure This! Includes a collection of mathematical challenges for middle school students and their families. Each challenge comes with a hint and the complete solution, along with related information and questions to think about. Visit the site at <http://www.figurethis.org>.

Federal Resources for Educational Excellence. Includes information and resources for parents and parent groups at <http://www.ed.gov/free>.

The Math Forum's Student Center. Features "Ask Dr. Math" and has weekly and monthly math challenges, Internet math hunts, and math resources organized by grade level. The forum is at <http://forum.swarthmore.edu>.

The National Council of Teachers of Mathematics. The Web site of this national professional organization of mathematics educators includes information on national standards in mathematics education, kindergarten through grade twelve. The site is at <http://www.nctm.org>.

The California Mathematics Council. The Web site of this statewide organization provides information and resources for parents. The site is at <http://cmc-math.org>.

Contact Information

For general information regarding the content standards and frameworks or the process for the state adoption of mathematics instructional materials, please contact the Curriculum Frameworks and Instructional Resources Division, California Department of Education (CDE), at (916) 319-0881.

For information regarding statewide testing in mathematics, please contact the Assessment Development and Administration Division, CDE, at (916) 319-0373.

For information regarding mathematics curriculum and instruction, including the California Department of Education publication *Literature for Science and Mathematics, Kindergarten Through Grade Twelve*, please contact the Science, Technology, Engineering, and Mathematics Office, CDE, at (916) 323-5847.

For information on family, parent, and community involvement, please contact the Improvement and Accountability Division, CDE, at (916) 319-0582.